The Extended Chiral Quark Model confronts QCD [*] [†]

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Abstract

We discuss the truncation of low energy effective action of QCD below the chiral symmetry breaking (CSB) scale, including all operators of dimensionality less or equal to 6 which can be built with quark and chiral fields. We perform its bosonization in the scalar, pseudoscalar, vector and axial-vector channels in the large-N_c and leading-log approximation. Constraints on the coefficients of the effective lagrangian are derived from the requirement of Chiral Symmetry Restoration (CSR) at energies above the CSB scale in the scalar-pseudoscalar and vector-axial-vector channels, from matching to QCD at intermediate scales, and by fitting some hadronic observables. In this truncation two types of pseudoscalar states (massless pions and massive Π-mesons), as well as a scalar, vector and axial-vector one arise as a consequence of dynamical chiral symmetry breaking. Their masses and coupling constants as well as a number of chiral structural constants are derived. A reasonable fit of all parameters supports a relatively heavy scalar meson (quarkonium) with the mass ~ 1 GeV and a small value of axial pion-quark coupling constant $g_A \simeq 0.55$.

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1. Introduction

The basic idea of the Extended Chiral Quark Model (ECQM) [1] consists in using the degrees of freedom which are relevant at each energy scale. It is built in terms of colored current quark fields $\bar{q}_i(x), q_i(x)$ with momenta restricted to be below the CSB scale $\Lambda_{CSB} \sim 1.3$ GeV, and colorless chiral fields $U(x) = \exp{(i\pi(x)/F_0)}$ which are $SU(N_F)$ matrices (herein $N_F = 2$) and which appear below Λ_{CSB}. The quarks are endowed with a 'constituent' mass M_0 multiplied by the chiral field $U(x)$ without manifestly breaking chiral symmetry. The information on modes with momenta larger than Λ_{CSB} as well as the effect of residual gluon interactions is contained in the coefficients of the effective lagrangian. The ECQM truncation of QCD effective action happens to be an extension of both the chiral quark model and the Nambu-Jona-Lasinio one.

The external sources are included into the QCD quark lagrangian in order to compute the correlators of corresponding quark currents

$$\hat{D} \equiv i\gamma_\mu(\partial_\mu + \bar{V}_\mu + \gamma_5 \bar{A}_\mu) + i(\bar{S} + i\gamma_5 \bar{P}), \tag{1}$$

where $\langle S \rangle = m_q$, matrix of current quark masses.

The low-energy effective lagrangian $\mathcal{L}_{ECQM}$ is built to be invariant under left and right $SU(2)$ rotations, of quark, chiral and external fields.

It is convenient to introduce the 'rotated', 'dressed' or 'constituent' quark fields

$$Q_L \equiv \xi q_L, \qquad Q_R \equiv \xi^\dagger q_R, \qquad \xi^2 \equiv U, \tag{2}$$

which transform nonlinearly under
$SU_L(2) \bigotimes SU_R(2)$ but identically for left and right quark components.

Changing to the 'dressed' basis implies the following replacements in the external vector, axial, scalar and pseudoscalar sources

$$\begin{aligned}
\bar{V}_\mu \to v_\mu &= \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger + \xi^\dagger\bar{V}_\mu\xi + \xi\bar{V}_\mu\xi^\dagger - \xi^\dagger\bar{A}_\mu\xi + \xi\bar{A}_\mu\xi^\dagger\right), \\
\bar{A}_\mu \to a_\mu &= \frac{1}{2}\left(-\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger - \xi^\dagger\bar{V}_\mu\xi + \xi\bar{V}_\mu\xi^\dagger + \xi^\dagger\bar{A}_\mu\xi + \xi\bar{A}_\mu\xi^\dagger\right), \\
\bar{\mathcal{M}} \to \mathcal{M} &= \xi^\dagger\bar{\mathcal{M}}\xi^\dagger.
\end{aligned} \tag{3}$$

In these variables the relevant part of ECQM action can be represented as,

$$\mathcal{L}_{ECQM} = \mathcal{L}_{ch} + \mathcal{L}_{\mathcal{M}} + \mathcal{L}_{vec}, \tag{4}$$

where $\mathcal{L}_{ch}$ accumulates the interaction of chiral fields and quarks in the chiral limit in the presence of vector and axial-vector external fields, $\mathcal{L}_{\mathcal{M}}$ extends the description for external scalar and pseudoscalar fields and, in particular, for massive quarks, $\mathcal{L}_{vec}$ contains operators generating meson states in vector and axial-vector channels.

In more detail[‡]

$$\begin{aligned}
\mathcal{L}_{ch} &= i\bar{Q}\left(\not{D} + M_0\right)Q - \frac{f_0^2}{4}\mathrm{tr}(a_\mu^2) \\
&\quad + \frac{G_{S0}}{4N_c\Lambda^2}\left(\bar{Q}_L Q_R + \bar{Q}_R Q_L\right)^2 - \frac{G_{P1}}{4N_c\Lambda^2}\left(-\bar{Q}_L\vec{\tau}Q_R + \bar{Q}_R\vec{\tau}Q_L\right)^2,
\end{aligned} \tag{5}$$

[‡]In (5) we have retained numerically the most important operators and only those four-quark vertices which induce a scalar isosinglet and pseudoscalar isotriplet meson states.

where

$$Q \equiv Q_L + Q_R, \qquad \not{D} \equiv \not{\partial} + \not{v} - \gamma_5 \tilde{g}_A \not{a}, \tag{6}$$

with the 'bare' pion decay constant f_0 and the 'bare' axial coupling $\tilde{g}_A \equiv 1 - \delta g_A$. These 'bare' contributions to the chiral effective lagrangian are renormalized after integration over low-energy quark

The massive part $\mathcal{L}_{\mathcal{M}}$ looks as follows

$$\begin{aligned}\mathcal{L}_{\mathcal{M}} &= i(\frac{1}{2} + \epsilon) \left(\bar{Q}_R \mathcal{M} Q_L + \bar{Q}_L \mathcal{M}^\dagger Q_R \right) \\ &+ i(\frac{1}{2} - \epsilon) \left(\bar{Q}_R \mathcal{M}^\dagger Q_L + \bar{Q}_L \mathcal{M} Q_R \right) \\ &+ \mathrm{tr} \left(c_0 \left(\mathcal{M} + \mathcal{M}^\dagger \right) + c_5 (\mathcal{M} + \mathcal{M}^\dagger) a_\mu^2 + c_8 \left(\mathcal{M}^2 + \left(\mathcal{M}^\dagger \right)^2 \right) \right), \end{aligned} \tag{7}$$

where the chiral couplings c_0, c_5, c_8 are 'bare', different from [2] and their physical values are controlled by the CSR rules (see below).

The chiral invariant quark self-interactions in the vector and axial-vector channels, $\mathcal{L}_{vec}$, are

$$\begin{aligned}\mathcal{L}_{vec} &= -\frac{G_{V1}}{4N_c \Lambda^2} \bar{Q} \vec{\tau} \gamma_\mu Q \bar{Q} \vec{\tau} \gamma_\mu Q - \frac{G_{A1}}{4N_c \Lambda^2} \bar{Q} \vec{\tau} \gamma_5 \gamma_\mu Q \bar{Q} \vec{\tau} \gamma_5 \gamma_\mu Q \\ &+ c_{10} \mathrm{tr} \left(U \bar{L}_{\mu\nu} U^\dagger \bar{R}_{\mu\nu} \right). \end{aligned} \tag{8}$$

Their inclusion leads to the appearance of vector and axial-vector isotriplet meson resonances . c_{10} is a 'bare' chiral coupling.

In total the effective action suitable for derivation of two-point correlators contains 13 parameters to be determined by matching to QCD: $M_0, \Lambda(\text{cutoff})$, the bare chiral constants $f_0, c_0, c_5, c_8, c_{10}$, the axial pion-quark coupling $\tilde{g}_A$, the mass asymmetry ϵ and the four-fermion coupling constants $G_{S0} \neq G_{P1}, G_{V1} \neq G_{A1}$.

2. Bosonization

We incorporate auxiliary fields Φ in the scalar and pseudoscalar channels, Σ, Π^a, and in the vector and axial-vector channel, $iW_\mu^{(\pm)a}$, and replace the four-fermion operators

$$\frac{G_C}{4N_c \Lambda^2} \bar{Q} \Gamma Q \bar{Q} \Gamma Q; \;\; \Gamma = 1; i\gamma_5 \tau^a; \gamma_\mu \tau^a; \gamma_5 \gamma_\mu \tau^a, \tag{9}$$

by

$$i\bar{Q} \Gamma \Phi Q + N_c \Lambda^2 \frac{\Phi^2}{G_C}; \quad C = S0; P1; V1; A1, \tag{10}$$

with an integration over new variables (see [1, 3]).

Due to vacuum polarization effects (quark loops) the auxiliary fields obtain kinetic terms and propagate, i.e. interpolate resonance states. One fulfills the confinement requirement for a finite number of resonances if one retains only that part of the quark loop which contains the leading logarithm of the cutoff Λ. In this approach one coherently neglects both the threshold part of quark loop ('continuum') and (the infinite number of) heavier resonance poles. This is supported by the large-N_c approximation which associates all momentum dependence in the bosonized effective action solely with meson resonances.

The actual value of constituent mass $< \Sigma >= \Sigma_0$ is described by the mass-gap equation

$$\frac{\Lambda^2}{G_{S0}}(\Sigma_0 - M_0) = -\frac{\Sigma_0^3}{4\pi^2}\ln\frac{\Lambda^2}{\Sigma_0^2} \equiv \Sigma_0^3 I_0. \tag{11}$$

Therefrom it is evident that the natural scale for the four-fermion interaction is given rather by Σ_0 than by Λ and it is useful to redefine the related coupling constants: $\bar{G}_C = G_C I_0 \frac{\Sigma_0^2}{\Lambda^2}$ for characterizing the weak coupling regime by $\bar{G}_C \ll 1$.

The leading-log part of the quark loop allows to find analytically both the mass spectrum and decay coupling constants of pions, heavy pions, scalar, vector and axial-vector resonances.
In particular, the physical axial coupling in pion-quark vertex is $g_A = \tilde{g}_A/(1+\bar{G}_A)$. The masses of vector mesons are evaluated to be

$$m_V^2 = \frac{6\Sigma_0^2}{\bar{G}_V}, \quad m_A^2 = 6\Sigma_0^2\frac{1+\bar{G}_A}{\bar{G}_A}. \tag{12}$$

Among others, their coupling constants to external vector fields are of main importance

$$f_V = \frac{N_c I_0}{6}; \qquad f_A = g_A f_V. \tag{13}$$

The pion decay constant can be found by taking into account the bare pion kinetic term (5)

$$F_0^2 = f_0^2 + N_c \Sigma_0^2 I_0 g_A \tilde{g}_A. \tag{14}$$

The pion mass is set by the quark condensate

$$C_q = \left(2c_0 + \frac{N_c}{4\pi^2}\Sigma_0^3 \ln\frac{\Lambda^2}{\Sigma_0^2}\right) \equiv -B_0 F_0^2, \tag{15}$$

according to the Gell-Mann-Oakes-Renner formula, $m_\pi^2 = 2m_q B_0$ and the masses of the u, d quarks are taken equal for simplicity.

Respectively the heavy Π mass is found to be

$$m_\Pi^2 = \frac{2\Sigma_0^2\tilde{g}_A}{\delta^2 g_A}(\frac{1}{\bar{G}_P} + 1), \quad \delta \equiv \frac{F_0}{f_0}. \tag{16}$$

The weak decay coupling constant for heavy Π meson reads

$$F_\Pi = F_0 d_1 \frac{m_\pi^2}{m_\Pi^2(0)}; \quad d_1 = \frac{\sqrt{1-\delta^2}}{\delta}\left(\frac{2\Sigma_0\epsilon}{\bar{G}_P g_A B_0} + 1\right). \tag{17}$$

The scalar meson mass is obtained in the form

$$m_\sigma^2 = 2\Sigma_0^2(\frac{1}{\bar{G}_S} + 3). \tag{18}$$

The matching to QCD yields further relations.

3. CSR matching

Let us exploit the constraints based on chiral symmetry restoration at QCD at high energies. We focus on two-point correlators of colorless quark currents

$$\Pi_C(p^2) = \int d^4x \exp(ipx) \langle T\left(\bar{q}\Gamma q(x) \bar{q}\Gamma q(0)\right)\rangle, \tag{19}$$

with the notations (9) and (10). In the chiral limit the scalar correlator and the pseudoscalar one coincide at all orders in perturbation theory and also at leading order in the non-perturbative O.P.E.[4] (see also [1, 5]).The same is true for the difference between the vector and axial-vector correlators [6]. As the above differences decrease rapidly with increasing momenta, one can expect that the lowest lying resonances included into ECQM will successfully saturate the constraints from CSB restoration.

In the scalar channel one obtains the following sum rules

$$c_8 + \frac{N_c \Sigma_0^2 I_0}{8\bar{G}_S} - \frac{4\epsilon^2 N_c \Sigma_0^2 I_0}{8\bar{G}_P} = 0, \tag{20}$$

$$Z_\sigma = Z_\pi + Z_\Pi, \qquad Z_\pi = 4B_0^2 F_0^2, \tag{21}$$

$$Z_\sigma m_\sigma^2 - Z_\Pi m_\Pi^2 \simeq 24\pi\alpha_s C_q^2 \sim 0, \tag{22}$$

where Z_σ, Z_π, Z_Π stand for the residues in resonance pole contributions in the scalar and pseudoscalar correlators. The first relation fixes unambiguously the bare constant c_8, the last one is essentially saturated by heavy pion parameters. As result of CSR sum rules one determines the chiral constant [2]

$$L_8 \simeq \frac{F_0^2}{16}\left(\frac{1}{m_\sigma^2} + \frac{1}{m_\Pi^2}\right), \tag{23}$$

as well as the asymmetry

$$2\epsilon = \frac{\bar{G}_P}{\bar{G}_S}\sqrt{\frac{g_A}{\tilde{g}_A}}\left(-\beta\sqrt{1-\delta^2} \pm \delta\sqrt{1-\beta^2}\right), \tag{24}$$

where $\beta \simeq \sqrt{1-(m_\sigma^2/m_\Pi^2)}$ and $\delta = f_0/F_0$.

In the vector channel one derives the relations

$$c_{10} = 0, \tag{25}$$

$$f_V^2 m_V^2 = f_A^2 m_A^2 + F_0^2, \tag{26}$$

$$f_V^2 m_V^4 = f_A^2 m_A^4, \tag{27}$$

where the two last ones represent the Weinberg sum rules. With the help of the first relation one obtains the chiral constant[2, 6]:

$$L_{10} = \frac{1}{4}\left(f_A^2 - f_V^2\right). \tag{28}$$

From the second one and eq.(13) we find

$$f_V^2 = \frac{F_0^2}{m_V^2(1-g_A^2\xi)} = \frac{N_c I_0}{6}, \qquad \xi = \frac{m_A^2}{m_V^2}, \tag{29}$$

and from the last one and eq.(13) it follows that

$$\xi = \frac{m_A^2}{m_V^2} = \frac{1}{g_A}. \tag{30}$$

The last QCD requirement we adopt concerns the CSR for the three-point correlator of one scalar and two axial-vector currents [1]. It determines eventually c_5 and the chiral constant L_5 [2]

$$c_5 \simeq 0; \qquad L_5 \simeq \frac{N_c \Sigma_0 I_0 g_A^2}{8 B_0 (1 + 3\bar{G}_S)}. \tag{31}$$

4. Fit and discussion

Let us specify the input parameters. As such we take $F_0 = 90$ MeV, $m_\pi^2 = 140$ MeV. We adopt [2, 7] $\hat{m}_q(1 \text{ GeV}) \simeq 6$ MeV, $B_0(1 \text{ GeV}) \simeq 1.5$ GeV, and engage the phenomenological value for the heavy pion mass $m_\Pi \simeq 1.3$ GeV [8]. We also take the vector and axial-vector meson masses, $m_\rho = 770$ MeV and $m_{a1} \simeq 1.2$ MeV, as known parameters. Then the parameter $\xi \simeq 2.4$.

Let us perform now an optimal fit applying in the vector channel only (25) and (26). For $m_\sigma \simeq 1$ GeV one finds $\beta \simeq 0.64$ and $L_8 \simeq 0.8 \times 10^{-3}$. For $g_A = 0.55$ one obtains $L_5 = 1.2 \times 10^{-3}$ (L_5, L_8 to be compared with [2]) and $\Sigma_0 \simeq 200$ MeV. Therefrom one derives that $\bar{G}_V \simeq 0.25$, $\bar{G}_A \simeq 0.2$, $\tilde{g}_A \simeq 0.66$. With these values, $I_0 \simeq 0.1$ and $\Lambda \simeq 1.3$ GeV. Then the bare pion coupling $f_0 \simeq 62$ MeV and for the rest of the parameters we find: $\delta \simeq 0.7$, $\bar{G}_S \simeq 0.11$, $\bar{G}_P \simeq 0.13$, and either $\epsilon \simeq 0.05$ or $\epsilon \simeq -0.51$. We see that indeed the four fermion coupling constants $\bar{G}_S$ and $\bar{G}_P$ as well as $\bar{G}_V$ and $\bar{G}_A$ are slightly different and their values $\ll 1$ signifying the weak coupling regime. We remark that for the value $g_A = 0.55$ the last sum rule (30) is imprecise: 2.4 vs. 1.8 .

The vector and axial vector couplings are $f_V = 0.22$ and $f_A = 0.12$ to be well compared with the experimental values [8] from the electromagnetic decays of ρ^0 and a_1 mesons. Two more predictions can be obtained: $F_\Pi = 0.8 \times 10^{-2} F_\pi$, $\quad F_\sigma = \frac{\sqrt{Z_\sigma}}{2B_0} = 1.6 F_0$. These constants are not yet experimentally measured.

Thus we have estimated all parameters of the ECQM effective lagrangian and made certain predictions. We conclude that the ECQM supplied with the CSR matching conditions proves to be a systematic way to describe hadron properties at low and intermediate energies starting from QCD.

An alternative scheme exists for modelling the QCD effective action at intermediate energies which is based on manifestly chiral invariant, quasilocal many-quark interaction[9]. Like the simple NJL model it exploits the hypotetical CSB mechanism due to strong attraction in scalar channels and yields a rather light scalar meson.

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D.Becirevic (Orsay): *Could you comment on why you did not use the last sum rule in the vector channel? How its inclusion may affect the scalar meson mass?*
A.A.Andrianov: *We, in fact, have performed the fit employing the sum rule (30). As a result, the mass of axial-vector meson comes out to be too low, 1 GeV or less, other parameters are changed slightly:* g_A *grows up and* Σ_0 *decreases. Thus we have disfavoured (30) not being satisfied with such a large discrepancy between physical and large-*N_c *values for* $a1$ *mass. As to the scalar meson its mass is governed by the scalar sum rules and the chiral constant* L_8 *and thereby is not affected by addition or neglection of (30).*